

Example 3 Simplify: a. $\frac{3}{\sqrt{5}}$ b. $\sqrt{\frac{7}{8}}$ c. $\frac{9\sqrt{3}}{\sqrt{24}}$ d. $\sqrt{3\frac{3}{7}} \cdot \sqrt{2\frac{1}{3}}$

Solution a. $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{(\sqrt{5})^2} = \frac{3\sqrt{5}}{5}$

b. $\sqrt{\frac{7}{8}} = \frac{\sqrt{7}}{\sqrt{8}} = \frac{\sqrt{7}}{2\sqrt{2}} = \frac{\sqrt{7}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{7 \cdot 2}}{2(\sqrt{2})^2} = \frac{\sqrt{14}}{4}$

c. $\frac{9\sqrt{3}}{\sqrt{24}} = \frac{9\sqrt{3}}{2\sqrt{6}} = \frac{9\sqrt{3}}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{9\sqrt{18}}{2(\sqrt{6})^2} = \frac{9\sqrt{9 \cdot 2}}{2 \cdot 6} = \frac{9 \cdot 3\sqrt{2}}{2 \cdot 6} = \frac{9\sqrt{2}}{4}$

d. $\sqrt{3\frac{3}{7}} \cdot \sqrt{2\frac{1}{3}} = \sqrt{\frac{24}{7}} \cdot \sqrt{\frac{7}{3}} = \sqrt{\frac{24}{7} \cdot \frac{7}{3}} = \sqrt{8} = 2\sqrt{2}$

Example 4 Multiply. Assume that all variables represent positive real numbers.

a. $3\sqrt{ab^2}(-2\sqrt{a})$ b. $\sqrt{r}(5 - \sqrt{r})$

Solution a. $3\sqrt{ab^2}(-2\sqrt{a}) = 3(-2)\sqrt{ab^2 \cdot a} = -6\sqrt{a^2b^2} = -6ab$

b. $\sqrt{r}(5 - \sqrt{r}) = 5\sqrt{r} - (\sqrt{r})^2 = 5\sqrt{r} - r$

Oral Exercises

Simplify.

1. $\sqrt{2} \cdot \sqrt{5}$

2. $\frac{\sqrt{32}}{\sqrt{2}}$

3. $\frac{\sqrt{45}}{\sqrt{5}}$

4. $4\sqrt{2} \cdot \sqrt{3}$

5. $\sqrt{3} \cdot \sqrt{6}$

6. $\sqrt{3} \cdot \sqrt{12}$

7. $\frac{\sqrt{18}}{\sqrt{3}}$

8. $\frac{\sqrt{48}}{\sqrt{2}}$

9. $\frac{\sqrt{5}}{\sqrt{15}}$

10. $\sqrt{\frac{2}{7}}$

Written Exercises

Simplify. Assume that all variables represent positive real numbers.

A 1. $5\sqrt{3} \cdot 2\sqrt{3}$

2. $4\sqrt{7} \cdot 2\sqrt{7}$

3. $\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{4}$

4. $\sqrt{5} \cdot \sqrt{5} \cdot \sqrt{9}$

5. $2\sqrt{5} \cdot \sqrt{7}$

6. $6\sqrt{2} \cdot \sqrt{5}$

7. $\sqrt{3} \cdot \sqrt{27}$

8. $\sqrt{5} \cdot \sqrt{20}$

9. $\sqrt{11} \cdot \sqrt{44}$

10. $\sqrt{7} \cdot \sqrt{35}$

11. $6\sqrt{72}$

12. $9\sqrt{242}$

13. $\sqrt{\frac{3}{8}} \cdot \sqrt{\frac{8}{3}}$

14. $\sqrt{\frac{4}{9}} \cdot \sqrt{\frac{18}{4}}$

15. $\sqrt{\frac{8}{11}} \cdot \sqrt{\frac{22}{32}}$

16. $\sqrt{\frac{7}{3}} \cdot \sqrt{\frac{3}{28}}$

17. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{8}{9}}$

18. $\sqrt{\frac{4}{5}} \cdot \sqrt{\frac{10}{36}}$

19. $\sqrt{3\frac{3}{5}} \cdot \sqrt{2\frac{1}{2}}$

20. $\sqrt{2\frac{2}{5}} \cdot \sqrt{1\frac{2}{3}}$

21. $\sqrt{\frac{10}{13}} \cdot \sqrt{\frac{1}{2}}$

22. $\sqrt{\frac{15}{11}} \cdot \sqrt{\frac{1}{3}}$

23. $\frac{6\sqrt{7}}{\sqrt{35}}$

24. $\frac{5\sqrt{48}}{\sqrt{39}}$

25. $3\sqrt{\frac{48}{9}}$

26. $7\sqrt{\frac{40}{49}}$

27. $\frac{14\sqrt{320}}{2\sqrt{5}}$

28. $\frac{15\sqrt{6}}{\sqrt{90}}$

29. $\sqrt{5}(\sqrt{5} - 2)$

30. $\sqrt{7}(6 - \sqrt{2})$

31. $(3\sqrt{2})(-2\sqrt{8})(3\sqrt{27})$

32. $(3\sqrt{5})(-\sqrt{10})(\sqrt{27})$

B 33. $(4\sqrt{a^2b})(3\sqrt{b})$

34. $(5\sqrt{mn^2})(-2\sqrt{m})$

35. $(-x\sqrt{x^2y})(y\sqrt{xy^2})$

36. $(-r\sqrt{r^2s})(-s\sqrt{r^2s})$

37. $\sqrt{m}(\sqrt{m^3} + 6)$

38. $\sqrt{x}(\sqrt{x^5} + 7)$

39. $(\sqrt{5x})(\sqrt{2x})(3\sqrt{10x^2})$

40. $(\sqrt{3a})(\sqrt{2a})(2\sqrt{6a^2})$

41. $\sqrt{32} \cdot \sqrt{2x} \cdot \sqrt{3x}$

42. $\sqrt{27} \cdot \sqrt{3n} \cdot \sqrt{5n}$

43. $(2\sqrt{5x})^2$

44. $2n(\sqrt{7n})^2$

45. $3x\sqrt{\frac{x}{y}} \cdot \sqrt{\frac{9x}{y}}$

46. $3q\sqrt{\frac{3q}{2r}} \cdot \sqrt{\frac{q}{r}}$

C 47. $\sqrt{3a}(\sqrt{12a} - 2\sqrt{8a^2})$

48. $\sqrt{6x}(\sqrt{3x} - 4\sqrt{8x^2})$

49. $3\sqrt{8m^3}(2\sqrt{2m} - 5\sqrt{8m^4})$

50. $2\sqrt{6x^3}(3\sqrt{8x} - 5\sqrt{3x^4})$

51. $(2\sqrt{3y^3})^3$

52. $(5\sqrt{2x^3})^3$

53. $(\sqrt{10xy})^3(x\sqrt{5x^3y} - y\sqrt{10xy^3})$

54. $(\sqrt{18ab})^3(a\sqrt{3a^2b} + b\sqrt{5ab^2})$

Rationalize the numerator.

Sample $\frac{\sqrt{5}}{\sqrt{7}} = \frac{\sqrt{5} \cdot \sqrt{5}}{\sqrt{7} \cdot \sqrt{5}} = \frac{(\sqrt{5})^2}{\sqrt{7 \cdot 5}} = \frac{5}{\sqrt{35}}$ ← no radical in numerator

55. $\frac{\sqrt{11}}{\sqrt{3}}$

56. $\frac{\sqrt{2}}{\sqrt{5}}$

57. $\frac{\sqrt{6}}{\sqrt{13}}$

58. $\frac{\sqrt{7}}{\sqrt{15}}$

59. $\frac{\sqrt{19}}{\sqrt{8}}$

Mixed Review Exercises**Solve.**

1. $x^2 = 169$

2. $2s^2 - 200 = 0$

3. $25z^2 - 1 = 15$

4. $\frac{1}{c} + \frac{1}{3} = \frac{1}{2}$

5. $\frac{5}{9} = \frac{25}{y}$

6. $\frac{8b - 5}{5b - 4} = \frac{13}{8}$

Simplify.

7. $19x + 2(3x - 4) + 5$

8. $12a + 7 - (8a - 17)$

9. $4(2b - 6) - 5(b - 4)$

10. $(-5c^2d)(-4cd^4)$

11. $-4m + 3 + 11m - 4$

12. $x(x + 2) + (x - 4)(2x + 1)$